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Influence of fiber's shape and size on overall elastoplastic property for micropolar composites

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Abstract

A general micromechanical method is developed for a micropolar composite with ellipsoidal fibers, where the matrix material is idealized as a micropolar material model. The method is based on a special micro–macro transition method, and the classical effective moduli for micropolar composites can be determined in an analytical way. The influence of both fiber's shape and size can be analyzed by the proposed method. The effective moduli, initial yield surface and effective nonlinear stress and strain relation for a micropolar composite reinforced by ellipsoidal fibers are examined, it is found that the prediction on the effective moduli and effective nonlinear stress and strain curves are always higher than those based on classical Cauchy material model, especially for the case where the size of fiber approaches to the characteristic length of matrix material. As expected, when the size of fiber is sufficiently large, the classical results (size-independence) can be recovered.

Keywords: Micropolar; Composite; Ellipsoidal fiber; Micromechanics; Plasticity; Effective property

1. Introduction

Classical micromechanics has been recognized a great success in predicting effective properties of composite material from its microstructural information (Nemat-Nasser and Hori, 1993; Hashin, 1983), it provides an efficient tool to tailor the composite with desired properties. However the classical micromechanics based on Cauchy material model fails to predict size effect well-observed for metal matrix composites

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(Kouzeli and Mortensen, 2002) and tiny structures (Fleck et al., 1993; Haque and Saif, 2003). In order to develop a systematic theory explaining the observed size-dependence of material properties, much efforts have been conducted to remedy the classical continue mechanics, usually by including high order terms or introducing new degrees of freedom in formulation, this leads to so called high order continue theory. Contrary to the classical (Cauchy) material model, the high order continue theory assumes that a material point can be still regarded as infinitely small, however there is microstructure inside of this point (Eringen, 1999). So there are two sets of variable to describe the motion of the material point in a high order continue theory, one is for the motion of inertia center of the point, the other is for the motion of the microstructure inside of this material point. With the specific constraint on the motion of microstructure, different classes of high order theories have been proposed, namely micromorphic, microstretch, micropolar, couple stress theories, which have been exposed in the monograph by Eringen (1999). The basic idea of the high order theory is to take into account the nonlocal effect due to the coarse microstructure presented in the material, and the Cauchy material model, in which any surface of a material element is assumed to transmit only force but not moment, is just the first-order approximation for the material with microstructure (Jasiuk and Ostoja-Starzewski, 1995). The other high order theories have also been proposed by different authors, for example, Aifantis (1984) proposed to incorporate the second gradient of deformation in constitutive relation, while the basic field equations keep unchanged. Based on the couple stress theory, Fleck and Hutchinson (1993) proposed a high order phenomenological theory to describe the size-dependence of plasticity, since then an intense research activity has been conducted on the high order theory (see for example, Gao et al., 1999; Huang et al., 2000; Forest et al., 2000).

The high order theory and homogenization technique have been used to explain the size-dependence for metal matrix composites and polycrystals. Wei (2001) employed the high order theory proposed by Fleck et al. (1993)) and finite element method to analyze the influence of particle size on the elastoplastic behavior of a composite material; using also finite element method, Chen and Wang (2002) idealized the matrix material as a micropolar material model and examine the influence of fiber's size on effective nonlinear stress and strain relation for a metal matrix composite. Zhu et al. (1997) used the high order theory proposed by Aifantis (1984) to study the effective plastic behavior of a particulate composite, the method is also based on finite element method. Analytical homogenization methods for a heterogeneous high order medium have also been proposed by Smyshlyaev and Fleck (1995) with strain gradient model and by Sharma and Dasgupta (2002) with a linear micropolar material model.

Recently Xun et al. (2004a,b), Liu and Hu (2005), Hu et al. (2005) have proposed an analytical micromechanical method to examine the classical elastic and plastic properties for particulate and long fiber composites. In this method, they idealize the matrix material with coarse microstructure as a micropolar material model. The proposed method can well describe the size effect observed for metal matrix composites, and at the same time it remains as an analytical formulation. The proposed method is based on micropolar Eshelby tensor proposed by Cheng and He (1995)) and on secant moduli concept, it can be considered as a natural extension of the classical secant moduli method based on the second-order stress moment in classical micromechanics (Qiu and Weng, 1992; Hu, 1996). This method can also be interpreted in a variation form of a generalized Ponte Castañeda's type (Ponte Castañeda, 1991; Hu et al., 2005), and it can in principle be applied for a more general composite in addition to particulate and long fiber (two-dimensional) composites. However effective properties for a more general micropolar composite with ellipsoidal fibers has not been addressed in a systematic way, especially by an analytical method. So the objective of this paper is to propose an analytical method to examine the effective elastic and plastic properties for a micropolar composite with aligned ellipsoidal fibers. The manuscript will be arranged as follows: a brief review on the basic element for micropolar theory and the microscopic to macroscopic transition method will be presented in Section 2, the theoretical analysis for evaluating the classical effective moduli and the nonlinear stress and strain relation of the composite will be given in Section 3, numerical examples will be provided in Section 4, followed by the conclusions.

2. Micropolar theory and micro–macro transition method

2.1. Micropolar theory

We are interested in the composite material where the coarse microstructure of matrix material must be taken into account due to the small size contrast between reinforced fiber and characteristic length of matrix material. In this case the matrix material is therefore idealized as a micropolar material model (Hu et al., 2005). Before proceeding, we will recall briefly basic elements for micropolar theory.

For a micropolar body without body force and couple, the governing equations are given by Eringen (1999) and Nowacki (1986):

$$\varepsilon_{ij} = u_{j,i} - e_{kij}\phi_k, \quad k_{ij} = \phi_{j,i} \tag{1a}$$

$$\sigma_{ij,i} = 0, \quad m_{ij,i} + e_{jik}\sigma_{ik} = 0 \tag{1b}$$

$$\sigma_{ji} = C_{jikl}\varepsilon_{kl} + B_{jikl}k_{kl}, \quad m_{ji} = B_{jikl}\varepsilon_{kl} + D_{jikl}k_{kl} \tag{1c}$$

where σ_{ij} and m_{ij} denote the stress and couple stress tensors, ε_{ij} and k_{ij} are the strain and torsion tensors, u_i and φ_i are the displacement and microrotation vectors, respectively. C_{ijkl}, B_{ijkl} and D_{ijkl} are the elasticity tensors of micropolar material, e_{ijk} is permutation tensor.

For a centrosymmetric and isotropic micropolar body, the elasticity tensors are specified as (Nowacki, 1986):

$$B_{jikl} = 0 \tag{2a}$$

$$C_{jikl} = \lambda\delta_{ij}\delta_{kl} + (\mu+\kappa)\delta_{jk}\delta_{il} + (\mu-\kappa)\delta_{ik}\delta_{jl} \tag{2b}$$

$$D_{jikl} = \alpha\delta_{ij}\delta_{kl} + (\beta+\gamma)\delta_{jk}\delta_{il} + (\beta-\gamma)\delta_{ik}\delta_{jl} \tag{2c}$$

where μ, λ are the classical Lame's constants and κ, γ, β, α are the new elastic constants introduced in micropolar theory. They constitute two set of moduli: μ, λ and κ have dimension of force per unit area, and γ, β, α have the dimension of force. δ_{ij} is the Kronecker delta. Due to the dimensional difference between the two sets of moduli, three intrinsic characteristic lengths can be defined for an isotropic elastic micropolar material, they can be defined as

$$l_1 = (\gamma/\mu)^{1/2}, \quad l_2 = (\beta/\mu)^{1/2}, \quad l_3 = (\alpha/\mu)^{1/2} \tag{3}$$

The constitutive equation (1c) can also be written in a simple form if we use $\sigma'_{(ij)}$, $\sigma_{\langle ij\rangle}$, $\sigma(\equiv\sigma_{ii})$ and $\varepsilon'_{(ij)}$, $\varepsilon_{\langle ij\rangle}$, $\varepsilon(\equiv\varepsilon_{ii})$ denoting separately the deviatoric symmetric, anti-symmetric and hydrostatic parts of the stress and strain tensors, and similar notations for the couple-stress and torsion tensors, the well-established elastic constitutive relations for a linear isotropic micropolar material can be rewritten as (Nowacki, 1986):

$$\sigma'_{(ij)} = 2\mu\varepsilon'_{(ij)}, \quad \sigma_{\langle ij\rangle} = 2\kappa\varepsilon_{\langle ij\rangle}, \quad \sigma = 3K\varepsilon \tag{4a}$$

$$m'_{(ij)} = 2\beta k'_{(ij)}, \quad m_{\langle ij\rangle} = 2\gamma k_{\langle ij\rangle}, \quad m = 3Nk \tag{4b}$$

and

$$K = \lambda + \frac{2}{3}\mu, \quad N = \alpha + \frac{2}{3}\beta \tag{5}$$

where K is the bulk modulus, N can be regarded as the corresponding stiffness measure for torsion, and symbols $()$ and $\langle\rangle$ in the subscript denote the symmetric and anti-symmetric parts of a tensor, respectively.

2.2. Micro–macro transition method

In order to define the effective properties for a micropolar composite, we have to consider a representative volume element (RVE). According to Xun et al. (2004a) and Liu and Hu (2005), the following special boundary conditions are prescribed on the RVE:

$$u_j = E_{(ij)}x_i, \quad \varphi_j = 0 \tag{6}$$

where $E_{(ij)}$ is symmetric and constant strain over RVE, u_j, φ_j are the components of displacement and rotation angle, respectively. This kind of boundary condition enables one to define the classical effective moduli of a micropolar composite, which relate the symmetric macroscopic stress and strain of the composite material, as will be explained below.

For any statically balanced local stress and couple stress fields (σ_{ij}, m_{ij}) and geometrically compatible local strain and torsion fields (ε_{ij}, k_{ij}), the volume average of the internal energy over the RVE is

$$\begin{aligned}\langle\sigma_{ij}\varepsilon_{ij} + m_{ij}k_{ij}\rangle &= \langle\sigma_{ij}(u_{j,i} - e_{kij}\varphi_k)\rangle + \langle m_{ij}\varphi_{j,i}\rangle = \frac{1}{V}\int_{\partial\mathrm{RVE}} \sigma_{ij}u_jn_i\,\mathrm{d}S + \frac{1}{V}\int_{\partial\mathrm{RVE}} m_{ij}\phi_jn_i\,\mathrm{d}S \\ &= E_{(mj)}\frac{1}{V}\int_{\partial\mathrm{RVE}} \sigma_{ij}x_mn_i\,\mathrm{d}S = E_{(mj)}\langle\sigma_{mj}\rangle\end{aligned} \tag{7}$$

It can also be shown that

$$\langle\varepsilon_{(ij)}\rangle = \frac{1}{2}\langle u_{i,j} + u_{j,i}\rangle = E_{(ij)} \tag{8}$$

where $\langle\cdot\rangle$ means the volume average of the said quantity over the RVE.

If only stress boundary condition is applied on the boundary of the RVE, that is

$$\sigma_{ij}n_i = \Sigma_{(ij)}n_i, \quad m_{ij}n_i = 0 \tag{9}$$

where n_i is the components of the outer normal of the RVE's boundary, it can also be shown that

$$\langle\sigma_{(ij)}\rangle = \Sigma_{(ij)}, \quad \langle\sigma_{ij}\varepsilon_{ij} + m_{ij}k_{ij}\rangle = \Sigma_{(mj)}\langle\varepsilon_{(mj)}\rangle \tag{10}$$

Finally the classical effective moduli (compliance) of a micropolar composite can be defined as

$$\langle\boldsymbol{\sigma} : \boldsymbol{\varepsilon} + \boldsymbol{m} : \boldsymbol{k}\rangle = \boldsymbol{E}^{\mathrm{sym}} : \boldsymbol{C}_{\mathrm{c}}^{\mathrm{sym}} : \boldsymbol{E}^{\mathrm{sym}} = \boldsymbol{\Sigma}^{\mathrm{sym}} : \boldsymbol{M}_{\mathrm{c}}^{\mathrm{sym}} : \boldsymbol{\Sigma}^{\mathrm{sym}} \tag{11}$$

where the superscript 'sym' means a symmetric tensor, $\boldsymbol{C}_{\mathrm{c}}^{\mathrm{sym}}, \boldsymbol{M}_{\mathrm{c}}^{\mathrm{sym}}$ relating the symmetric stress and strain by $\boldsymbol{E}^{\mathrm{sym}} = \boldsymbol{M}_{\mathrm{c}}^{\mathrm{sym}} : \boldsymbol{\Sigma}^{\mathrm{sym}}$ or $\boldsymbol{\Sigma}^{\mathrm{sym}} = \boldsymbol{L}_{\mathrm{c}}^{\mathrm{sym}} : \boldsymbol{E}^{\mathrm{sym}}$, which are called classical moduli and compliance of the micropolar composite in the following discussion.

In the following section, we will propose an analytical method for evaluating the classical effective modulus (or compliance) for a general micropolar composite with aligned ellipsoidal fibers.

3. Theoretical formulation

3.1. Eshelby tensor for an ellipsoidal inclusion

Following Ma and Hu (submitted), consider an inclusion Ω in an infinite centrosymmetric and isotropic micropolar material, characterized by moduli $\boldsymbol{C}_0$ and $\boldsymbol{D}_0$, a uniform asymmetric eigenstrain $\boldsymbol{\varepsilon}^*$ and an eigentorsion $\boldsymbol{k}^*$ are prescribed in the inclusion. Here the inclusion means its material constants are the same as the surrounding matrix (Mura, 1982). According to Cheng and He (1995) and Ma and Hu (submitted), the induced strain and torsion by the prescribed eigenstrain and eigentorsion can be written as

$$\boldsymbol{\varepsilon}(\mathbf{x}) = \boldsymbol{S}(\mathbf{x}) : \boldsymbol{\varepsilon}^* + \boldsymbol{L}(\mathbf{x}) : \boldsymbol{k}^* \tag{12a}$$

$$\boldsymbol{k}(\mathbf{x}) = \widehat{\boldsymbol{S}}(\mathbf{x}) : \boldsymbol{\varepsilon}^* + \widehat{\boldsymbol{L}}(\mathbf{x}) : \boldsymbol{k}^* \tag{12b}$$

where the tensors $\boldsymbol{S}$, $\hat{\boldsymbol{S}}$, $\boldsymbol{L}$ and $\widehat{\boldsymbol{L}}$ are called micropolar Eshelby tensors, and their expressions for a general ellipsoidal inclusion are listed in Appendix A. Unlike the Eshelby tensor for the classical material (Cauchy material model), the micropolar Eshelby tensors are not uniform even inside of an ellipsoidal inclusion. However the numerical computation shows that their variation in the ellipsoidal inclusion is not significant (Ma and Hu, submitted). So in the following, the averages of the micropolar Eshelby tensors over the ellipsoidal inclusion will be used to determine the classical effective property for a micropolar composite. It can be demonstrated that for a general ellipsoidal inclusion, the following relations for the average micropolar Eshelby tensors hold (Ma and Hu, submitted)

$$\langle \boldsymbol{L} \rangle_{\mathrm{I}} = 0, \quad \langle \widehat{\boldsymbol{S}} \rangle_{\mathrm{I}} = 0 \tag{13}$$

where $\langle \cdot \rangle_{\mathrm{I}}$ means the volume average of the said quantity over the inclusion domain. Eq. (13) means that a uniform eigenstrain only induces a nonzero average strain and a uniform eigentorsion produces only a nonzero average torsion for a general ellipsoidal inclusion. The average micropolar Eshelby relations (Eq. (12)) are uncoupled. That is

$$\langle \boldsymbol{\varepsilon} \rangle_{\mathrm{I}} = \langle \boldsymbol{S} \rangle_{\mathrm{I}} : \boldsymbol{\varepsilon}^*, \quad \langle \boldsymbol{k} \rangle_{\mathrm{I}} = \langle \widehat{\boldsymbol{L}} \rangle_{\mathrm{I}} : \boldsymbol{k}^* \tag{14}$$

These two micropolar Eshelby tensors $\langle \boldsymbol{S} \rangle_{\mathrm{I}}$ and $\langle \widehat{\boldsymbol{L}} \rangle_{\mathrm{I}}$ will be computed with the expressions given in Appendix A, the detailed exposition of the micropolar Eshelby tensors for the ellipsoidal inclusion can be found in the reference by Ma and Hu (submitted).

For an inhomogeneity problem, which the material property ($\boldsymbol{C}_1$, $\boldsymbol{D}_1$) of the inclusion is different from the surrounding matrix ($\boldsymbol{C}_0$, $\boldsymbol{D}_0$), the equivalent inclusion method widely used in the classical micromechanics cannot be applied exactly for a micropolar material (Xun et al., 2004a), since the strain and torsion in the ellipsoidal inhomogeneity are not uniform. To simplify the problem, the average equivalent method will be used in this paper to determine the strain and torsion in the inhomogeneity, which can be written as

$$\boldsymbol{C}_1(\boldsymbol{E}_0 + \langle \boldsymbol{\varepsilon} \rangle_{\mathrm{I}}) = \boldsymbol{C}_0(\boldsymbol{E}_0 + \langle \boldsymbol{\varepsilon} \rangle_{\mathrm{I}} - \bar{\boldsymbol{\varepsilon}}^*) \tag{15a}$$

$$\boldsymbol{D}_1(\boldsymbol{K}_0 + \langle \boldsymbol{k} \rangle_{\mathrm{I}}) = \boldsymbol{D}_0(\boldsymbol{K}_0 + \langle \boldsymbol{k} \rangle_{\mathrm{I}} - \bar{\boldsymbol{k}}^*) \tag{15b}$$

where $\bar{\boldsymbol{\varepsilon}}^*, \bar{\boldsymbol{k}}^*$ are average eigenstrain and eigentorsion; $\boldsymbol{E}_0$,$\boldsymbol{K}_0$ are the remote applied strain and torsion respectively.

It is shown by Xun et al. (2004a) that for a cylindrical inhomogeneity the average stress determined by the average equivalent method agrees well with the exact results. However, since there is no exact solution for a general ellipsoidal inclusion, so Eq. (15) remains as a hypothesis.

3.2. Classical effective moduli of micropolar composite

For a micropolar composite with many aligned ellipsoidal fibers (the properties of the matrix and fiber are noted respectively by $\boldsymbol{C}_0$, $\boldsymbol{D}_0$ and $\boldsymbol{C}_1$, $\boldsymbol{D}_1$), the volume fraction of fiber is noted by f. The concept of Mori–Tanaka's method (Mori and Tanaka, 1973) in classical micromechanics will be employed to consider the interaction between the fibers. Now a single fiber is placed into the infinite micropolar matrix material under remote loading $\boldsymbol{E}_0$, $\boldsymbol{K}_0$, which are the unknown average strain and torsion of the micropolar matrix in the actual composite. So the average equivalent inclusion method (Eq. (15)) can be applied for each fiber, together with Eq. (14) ($\boldsymbol{\varepsilon}^*$, $\boldsymbol{k}^*$ being replaced by $\bar{\boldsymbol{\varepsilon}}^*, \bar{\boldsymbol{k}}^*$), the average stress (strain) and couple stress (torsion) in each fiber can be related to the remote applied load $\boldsymbol{E}_0$, $\boldsymbol{K}_0$.

Since we are interested in the classical effective moduli of the micropolar composite, according to the micro–macro transition method explained in Section 2, only the symmetric parts of the strain relations in Eqs. (8) and (9) will be used. So we have

$$\boldsymbol{C}_1^{\text{sym}} : (\boldsymbol{E}_0^{\text{sym}} + \langle \boldsymbol{\varepsilon}^{\text{sym}} \rangle_{\text{I}}) = \boldsymbol{C}_0^{\text{sym}} : (\boldsymbol{E}_0^{\text{sym}} + \langle \boldsymbol{\varepsilon}^{\text{sym}} \rangle_{\text{I}} - \bar{\boldsymbol{\varepsilon}}^{*\text{sym}}) \tag{16}$$

$$\langle \boldsymbol{\varepsilon}^{\text{sym}} \rangle_{\text{I}} = \langle \boldsymbol{S}^{\text{sym}} \rangle_{\text{I}} : \bar{\boldsymbol{\varepsilon}}^{*\text{sym}} \tag{17}$$

where $S_{ijmn}^{\text{sym}} = (S_{ijmn} + S_{ijnm} + S_{jimn} + S_{jinm})/4$ for a fourth-order tensor and $\varepsilon_{ij}^{\text{sym}} = (\varepsilon_{ij} + \varepsilon_{ji})/2$ for a second-order tensor.

With help of the micro–macro transition principle and Mori–Tanaka's method, it can be shown that

$$\boldsymbol{E}^{\text{sym}} = (1-f)\boldsymbol{E}_0^{\text{sym}} + f(\boldsymbol{E}_0^{\text{sym}} + \langle \boldsymbol{\varepsilon}^{\text{sym}} \rangle_{\text{I}}) \tag{18}$$

and using

$$\boldsymbol{\Sigma}^{\text{sym}} = (1-f)\boldsymbol{\Sigma}_0^{\text{sym}} + f(\boldsymbol{\Sigma}_0^{\text{sym}} + \langle \boldsymbol{\sigma}^{\text{sym}} \rangle_{\text{I}}) \tag{19}$$

the classical effective modulus of the composite can be calculated by following exactly the same method as the classical Mori–Tanaka's method (see for example, Hu and Weng, 2000), and the classical effective compliance of the micropolar composite (here we assume the fiber is classical material) can be finally derived in an analytical from as

$$\boldsymbol{M}_{\text{c}}^{\text{sym}} = \boldsymbol{M}_0^{\text{sym}} + f\left[(\boldsymbol{M}_1 : (\boldsymbol{M}_0^{\text{sym}})^{-1} - \boldsymbol{I})^{-1} + (1-f)(\boldsymbol{I} - \langle \boldsymbol{S}^{\text{sym}} \rangle_{\text{I}})\right]^{-1} : \boldsymbol{M}_0^{\text{sym}} \tag{20}$$

where $\boldsymbol{M}_{\text{c}}^{\text{sym}}, \boldsymbol{M}_0^{\text{sym}}$ and $\boldsymbol{M}_1$ are the inverses of the tensor $\boldsymbol{C}_{\text{c}}^{\text{sym}}, \boldsymbol{C}_0^{\text{sym}}$ and $\boldsymbol{C}_1$, separately. $\boldsymbol{I}$ is the fourth-order unit tensor.

Eq. (20) has the same form as that predicted by Mori–Tanaka's method for a classical composite, and for the classical composite, $\langle \boldsymbol{S}^{\text{sym}} \rangle_{\text{I}}$ is replaced by the classical Eshelby tensor of an ellipsoidal inclusion. The size effect of microstructure is taken into account through the average micropolar Eshelby tensor $\langle \boldsymbol{S}^{\text{sym}} \rangle_{\text{I}}$ in Eq. (20). It can be shown that when the fiber's size (diameter) is sufficiently large, $\langle \boldsymbol{S}^{\text{sym}} \rangle_{\text{I}}$ will be reduced to the classical Eshelby tensor (Ma and Hu, submitted), so the classical results can be recovered.

3.3. Yield surface of micropolar composite

In this section, the initial yield surface of a micropolar composite will be examined. It is assumed that when the average equivalent stress of the micropolar matrix reaches its initial yield stress σ_{y}, then the composite starts to yield. That is

$$\langle \sigma_{\text{eff}} \rangle_0 = \sigma_{\text{y}} \tag{21}$$

where $\langle \cdot \rangle_0$ means the volume average of the said quantity over the matrix, and σ_{eff} is an effective stress for the micropolar matrix, which is defined as for a micropolar material by (Hu et al., 2005)

$$\sigma_{\text{eff}}^2 = \frac{3}{2}\sigma'_{(ij)}\sigma'_{(ij)} + \frac{3}{2}l^2(m'_{(ij)}m'_{(ij)} + m_{\langle ij \rangle}m_{\langle ij \rangle}) \tag{22}$$

For simplification, we assume that the elastic characteristic lengths of the matrix material to be equal: $l_1 = l_2 = l_3 = l$.

Let the generalized yield surface of the micropolar composite be specified by

$$\Phi(\langle \sigma_{\text{eff}} \rangle_0, \sigma_{\text{y}}) = \langle \sigma_{\text{eff}} \rangle_0 - \sigma_{\text{y}} = 0 \tag{23}$$

To compute $\langle\sigma_{\text{eff}}\rangle_0$ in an analytical way for a general micropolar composite, it is further assumed that $\langle\sigma_{\text{eff}}\rangle_0 \approx \sqrt{\langle\sigma^2_{\text{eff}}\rangle_0}$, this assumption is widely made for classical micromechanics of plasticity (Hu, 1996; Ponte Castañeda, 1991).

With help of the perturbation method developed for micropolar composites (Liu and Hu, 2005), the second-order moment of stress and couple stress of the matrix material defined by Eq. (22) can then be evaluated analytically, finally the generalized yield surface of the micropolar composite can be expressed as

$$\frac{3}{1-f}\boldsymbol{\Sigma}^{\text{sym}} : \boldsymbol{Q} : \boldsymbol{\Sigma}^{\text{sym}} + \sigma_{\text{y}}^2 = 0 \tag{24}$$

where

$$\boldsymbol{Q} = \mu_0^2 \frac{\partial \boldsymbol{M}_{\text{c}}^{\text{sym}}}{\partial \mu_0} + \frac{1}{l^2}\left(\beta_0^2 \frac{\partial \boldsymbol{M}_{\text{c}}^{\text{sym}}}{\partial \beta_0} + \gamma_0^2 \frac{\partial \boldsymbol{M}_{\text{c}}^{\text{sym}}}{\partial \gamma_0}\right)$$

The analytical expression for $\boldsymbol{M}_{\text{c}}^{\text{sym}}$ is given by Eq. (20).

3.4. Nonlinear stress and strain relation of micropolar composite

When the applied load is larger than the initial yield stress of the composite, plastic deformation of the matrix will take place. In order to model the weakened constraint power of the plastic matrix on the fiber, the secant moduli method based on second-order stress and couple stress moment will be utilized (Liu and Hu, 2005).

Supposed the stress potential w of the micropolar matrix to be the following form:

$$w = w_0(\sigma_{\text{eff}}) + \frac{1}{6\kappa_0}\sigma^2_{\langle e\rangle} + \frac{1}{18K_0}\sigma^2 + \frac{1}{18N_0}m^2 \tag{25}$$

where

$$\sigma_{\langle e\rangle} = \sqrt{\frac{3}{2}\sigma_{\langle ij\rangle}\sigma_{\langle ij\rangle}}, \quad w_0(\sigma_{\text{eff}}) = \frac{\sigma^2_{\text{eff}}}{6\mu_0} + \frac{n}{n+1}\frac{1}{H^{1/n}}(\sigma_{\text{eff}} - \sigma_{\text{y}})^{\frac{n+1}{n}}$$

and n is the strain hardening exponent, H is the hardening modulus of the matrix material.

The secant moduli of the matrix material can be defined through the constitutive relation, this leads to the following secant quantities for the matrix material (Hu et al., 2005):

$$\mu_0^{\text{s}} = \frac{1}{(1/\mu_0) + 3[(\sigma_{\text{eff}} - \sigma_{\text{y}})/H]^{1/n}/\sigma_{\text{eff}}}, \quad \kappa_0^{\text{s}} = \kappa, \quad K_0^{\text{s}} = K_0 \tag{26a}$$

$$\beta_0^{\text{s}} = l^2\mu_0^{\text{s}}, \quad \gamma_0^{\text{s}} = l^2\mu_0^{\text{s}}, \quad N_0^{\text{s}} = N_0 \tag{26b}$$

where the superscript s means the secant quantities.

With the secant moduli of the matrix defined in Eqs. (26a) and (26b), we can follow exactly the same idea of the secant moduli method developed for a classical composite (see for example, Qiu and Weng, 1992; Hu, 1996) to compute the nonlinear stress and strain relation of a micropolar composite. This can be explained by the following procedure: for any given macroscopic stress $\boldsymbol{\Sigma}^{\text{sym}}$, at which the matrix has entered into plastic state, for a tested average effective stress of the matrix $\langle\sigma_{\text{eff}}\rangle_0$ ($>\sigma_{\text{y}}$), the secant moduli of the matrix can be evaluated by Eq. (26). We consider a linear comparison composite, it has the same microstructure and fiber's property as the actual composite, however its matrix has the secant moduli of the actual matrix in the nonlinear composite. The compliance tensors $\boldsymbol{M}_{\text{c}}^{\text{sym}}$ of this linear comparison composite can be determined from Eq. (20). The average effective stress of the micropolar matrix for the linear comparison composite can then be evaluated with the expression of $\boldsymbol{M}_{\text{c}}^{\text{sym}}$, this provides an equation to determine for a given applied load the Σ^{sym} corresponding $\langle\sigma_{\text{eff}}\rangle_0$. The moduli of the linear comparison composite are interpreted as the secant moduli of the actual composite. By repeating Σ^{sym}, the nonlinear stress and strain curves of the micropolar composite material can then be established.

In the following, we will examine through some numerical examples the size-dependence of the effective properties for a micropolar composite with aligned ellipsoidal fibers.

4. Numerical applications

A metal matrix composite SiC/Al is taken to be the sample material, the material constants are $\mu_0 = 26$ GPa, $\lambda_0 = 50$ GPa, $\kappa_0 = 13$ GPa, $l = 10$ μm for matrix and $\mu_1 = 209$ GPa, $\lambda_1 = 108$ GPa for the fiber material, the other parameters will be specified when used.

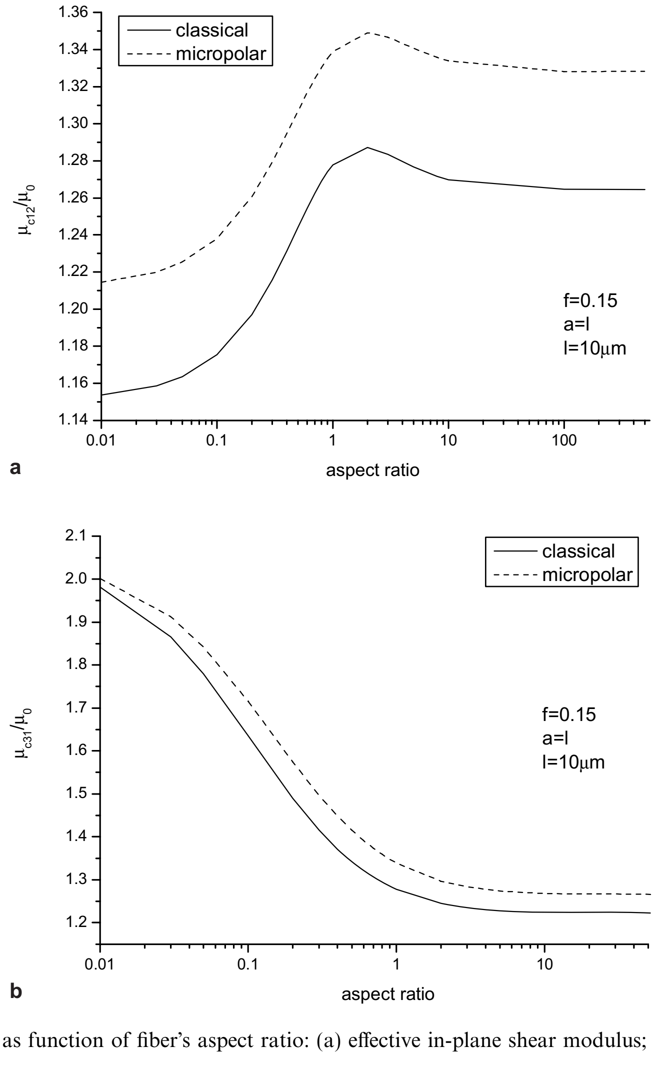

Fig. 1. Effective shear moduli as function of fiber's aspect ratio: (a) effective in-plane shear modulus; (b) effective out-of-plane shear modulus.

4.1. Elastic moduli

Fibers are assumed to be aligned along x_3 direction, the modulus in the plane x_1–x_2 is called in-plane modulus and the others are called out-of-plane modulus in the following. We use the radius of the ellipsoid a to characterize its size, and its shape is described by its aspect ratio. The predicted effective in-plane shear modulus μ_{c12} and out-of-plane shear modulus μ_{c31} as function of fiber's aspect ratio are shown in Fig. 1a and b, respectively. The fiber's size is set to be $a = l$, and its volume fraction is $f = 0.15$. For comparison, the effective shear moduli for the composite with a classical matrix is also included. It is found that the predicted effective shear moduli (in-plane and out-of-plane) of the micropolar composite are slightly higher than those for the classical composite, the dependence on the fiber's aspect ratio is the same for these two models.

The aspect ratio of the fiber is now kept to be 10, the predicted effective shear modulus as function of fiber size is illustrated in Fig. 2a and b for two volume fractions of the fiber $f = 0.15$ and $f = 0.3$,

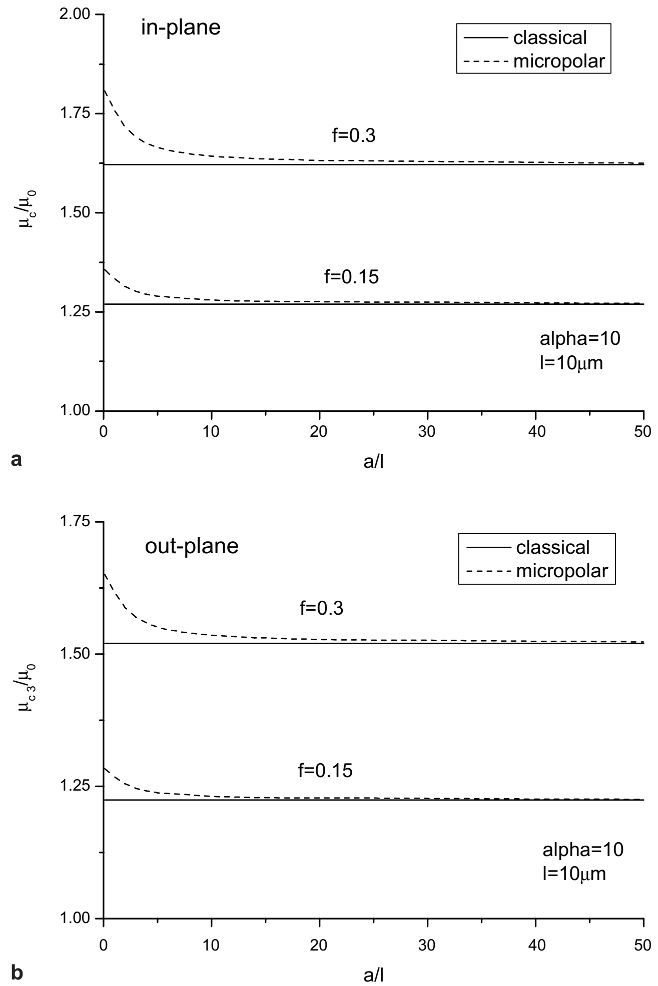

Fig. 2. Effective shear moduli as function of fiber's size with a fixed aspect ratio 10: (a) effective in-plane shear modulus; (b) effective out-of-plane shear modulus.

respectively, the classical predictions are also included. As for the particulate and fiber composites (Xun et al., 2004b; Liu and Hu, 2005), when the size of the fiber is comparable or less than the intrinsic length

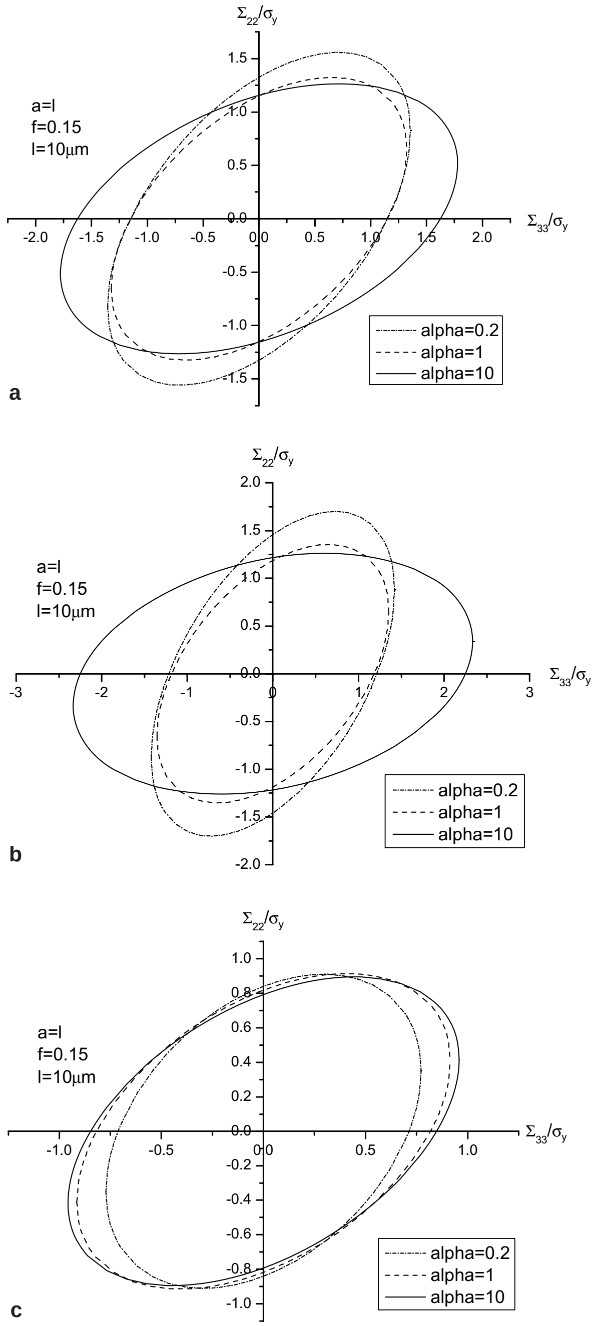

Fig. 3. Yield surfaces in Σ_{33}–Σ_{22} stress-space for three different fiber properties and for three different fiber's aspect ratios 0.2, 1, 10: (a) common fiber; (b) rigid fiber; (c) voids.

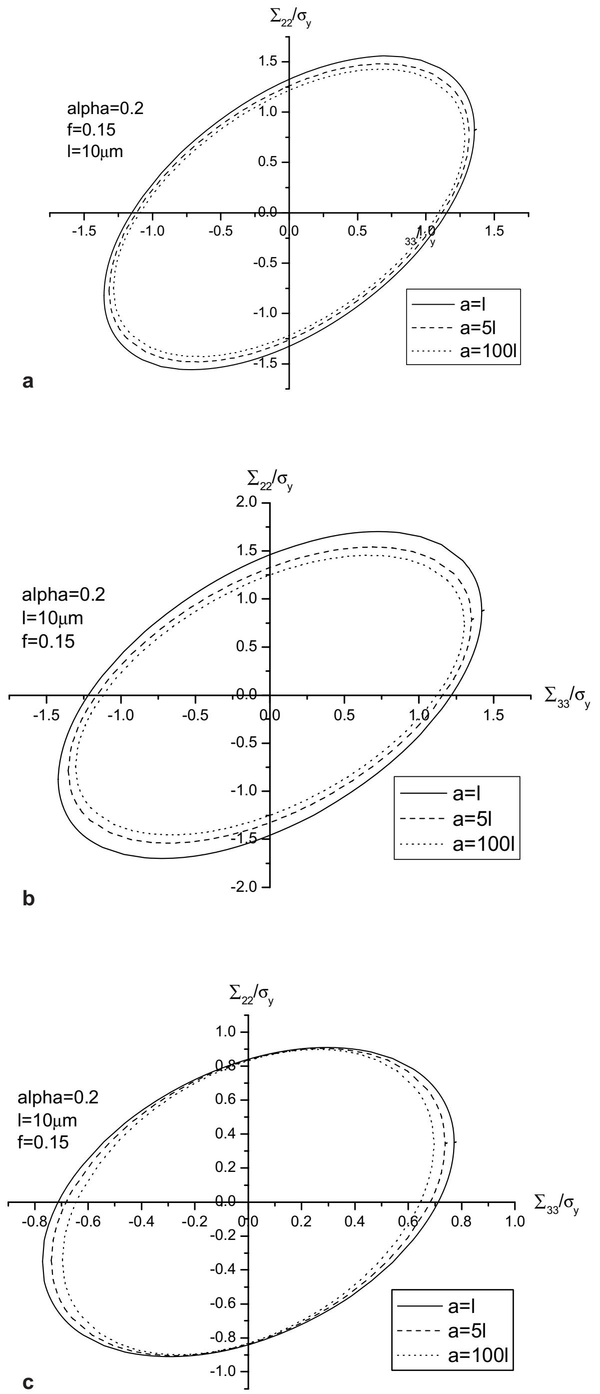

Fig. 4. Yield surfaces in Σ_{33}–Σ_{22} stress-space of different fiber's properties for three different fiber sizes: $a = l$, $a = 5l$, $a = 100l$: (a) common fiber; (b) rigid fiber; (c) voids.

of the matrix material, the size effect of the fiber on the effective shear moduli of the composite is more pronounced. When the size of the fiber is much larger than the intrinsic length of the matrix material, classical results can be recovered, as expected.

4.2. Initial yield surface

The material constants are the same as in Section 4.1, the initial yield stress of the micropolar matrix is $\sigma_y = 250$ MPa. Three kinds of fiber's properties are examined: one is a common metal matrix composite, the fiber's material constants have been given in Section 4.1, another is a rigid fiber composite, the third one is a voided material. Fig. 3a–c shows the yield surfaces in Σ_{33}–Σ_{22} stress-space for the three types of the fiber and for three different fiber's aspect ratios 0.2, 1, 10. The fiber's size and volume fraction is and

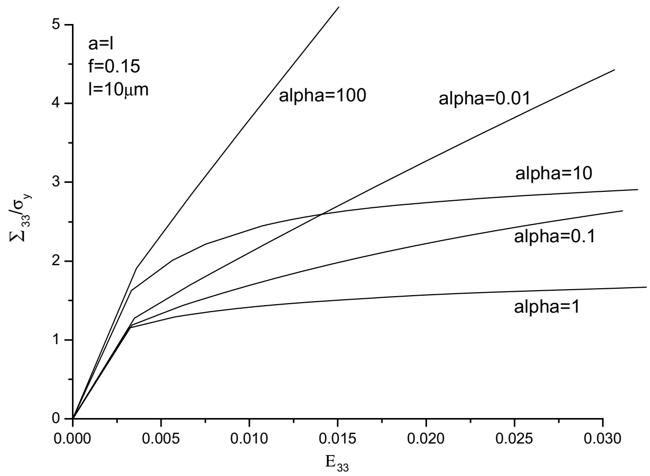

Fig. 5. Stress and strain relations of a metal matrix composite for different fiber's aspect ratios 0.01, 0.1, 1, 10, 100.

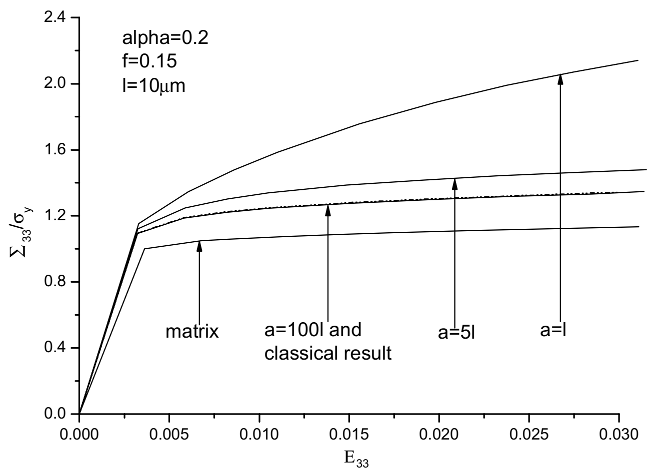

Fig. 6. Stress and strain relations of a metal matrix composite for three different fiber's sizes: $a = l$, $a = 5l$, $a = 100l$.

$f = 0.15$ respectively. As predicted by the classical method, the fiber's shape has significant influence on the initial yield surface of the composite, anisotropic yielding will be generated if the aspect ratio of the fiber is not equal to unity.

Fig. 4a–c shows the yield surfaces in Σ_{33}–Σ_{22} stress-space for three different fiber sizes:, $a = 5l$, $a = 100l$ respectively, while the fiber's aspect ratio is taken to be 0.2 and $f = 0.15$. It is found that when the fiber's size decreases, the yield surface of the composite is slightly enlarged. For rigid fiber, the size effect is more pronounced.

4.3. Nonlinear stress and strain relation

For the composite with aligned fibers, a tensile loading is applied along fiber's direction. The material constants are same as in Section 4 for a common metal matrix composite. The plastic parameters of the matrix are: $\sigma_y = 250$ MPa, $h = 173$ MPa and $n = 0.455$.

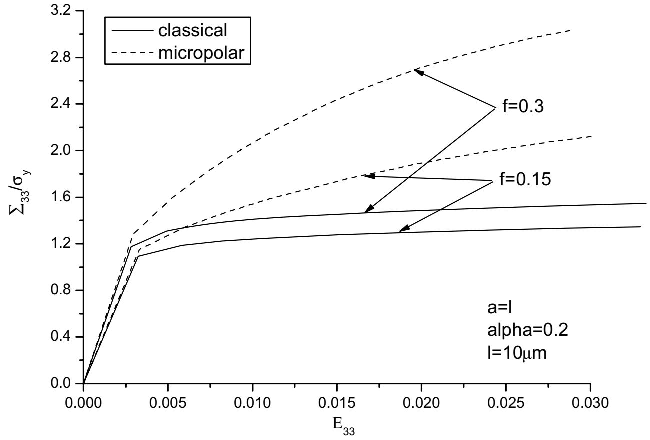

Fig. 7. Comparison with classical Cauchy model for a metal matrix composite for two fiber's volume fractions $f = 0.15$, 0.3.

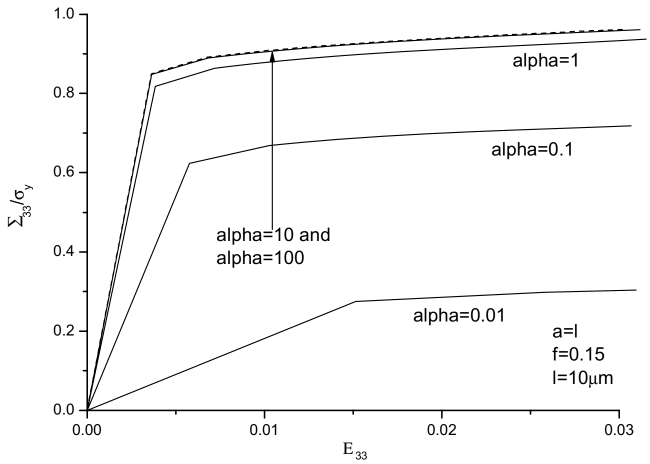

Fig. 8. Stress and strain relations for a voided material at different fiber aspect ratios: 0.01, 0.1, 1, 10, 100.

The influence of fiber's aspect ratio on the stress and strain curves for a common composite (metal matrix composite) is illustrated in Fig. 5, fibers with aspect ratio 0.01, 0.1, 1, 10,100 are examined. The fiber size is set to be $a = l$ and its volume fraction is $f = 0.15$. It is found that fiber's shape has a significant influence on the stress and strain relation for the composite, the same as for a Cauchy composite. Now fixed the fiber's aspect ratio to be 0.2, the other material parameters remain unchanged, the stress and strain curves of the composite for three different fiber sizes: $a = l$, $a = 5l$ and $a = 100l$ are examined, which are shown in Fig. 6. The predictions for the composite with the classical matrix and un-reinforced matrix are also included for comparison. The computed results show that the influence of fiber's size is also important for a common metal matrix composite, especially when the fiber's size approaches to the characteristic length of the matrix material. However when the fiber's size is large, the predicted results by the current method is reduced to the classical one as it should be. The comparison with the classical micromechanics is also

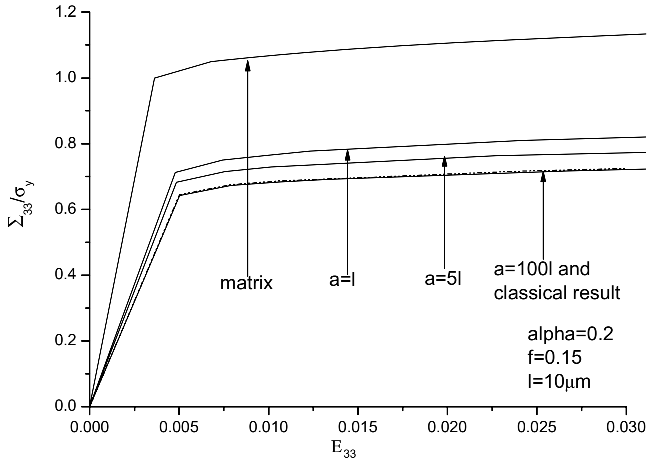

Fig. 9. Stress and strain relations for a voided material at three different fiber sizes: $a = l$, $a = 5l$, $a = 100l$.

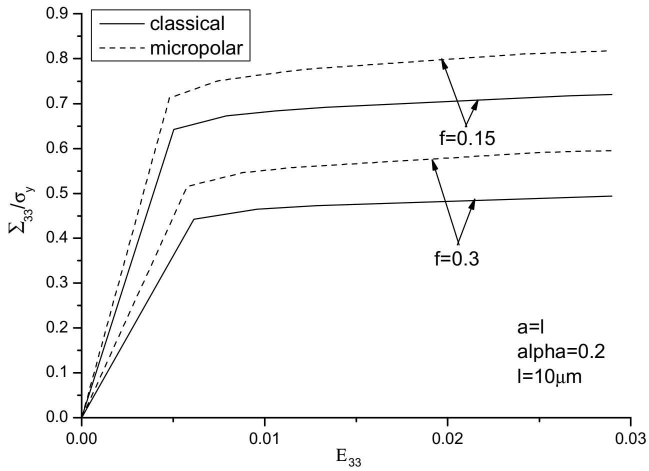

Fig. 10. Comparison with classical Cauchy method for a voided material at different volume fractions $f = 0.15$, 0.3.

conducted for two fiber's volume fractions $f = 0.15$, 0.3, and the aspect ratio of the fiber is 0.2 and $a = l$, the results are shown in Fig. 7. It is found that the prediction by the micropolar model is always higher than that by the Cauchy model, significant hardening behavior can be produced by the proposed micropolar model.

The effective stress and strain relations for a voided material are also examined, the influence of void's aspect ratio on the stress and strain relations is shown in Fig. 8, as for common metal matrix composite the shape of the voids has great influence for oblate voids, much less for prolate voids. The influence of the void's size on the stress and strain relation is also illustrated in Fig. 9. Compared to the common metal matrix composite, the effect of void's size on the stress and strain relation is much less significant. The comparison with the classical results (based on Cauchy material model) is also given in Fig. 10, as for the common metal matrix composite, the prediction by the current method is always higher than that by the classical method.

5. Conclusions

A micromechanical method is proposed for a micropolar composite with ellipsoidal fibers to examine the influence of the both fiber's shape and size on the effective elastic and plastic properties of composite materials. The method is based on a special micro–macro transition method and the micropolar Eshelby tensors for a general ellipsoidal inclusion, the classical effective moduli for a micropolar composite are determined analytically by this method. The nonlinear stress and strain relation of micropolar composites are established by the secant moduli method based on second-order stress and couple stress moment. With this method, the influence of the both fiber's shape and size can be analyzed in a simple analytical way. The effective moduli, initial yield surface and the effective plastic stress and strain relation for a micropolar composite reinforced by ellipsoidal fibers are analyzed in details. The results show that the prediction based on the micropolar material model for the effective moduli and effective nonlinear stress and strain curves are always higher than those based on classical Cauchy material model, especially for small size of fiber. When the size of fiber is sufficiently large, the classical results (Cauchy material model) can be found.

Acknowledgement

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Appendix A. Eshelby tensor for an ellipsoidal inclusion (Ma and Hu, submitted)

The expressions of micropolar Eshelby tensors are given by (Cheng and He, 1995; Ma and Hu, submitted):

$$S_{mnji}(\mathbf{x}) = I^{S}_{nji,m}(\mathbf{x}) + I_{nji,m}(\mathbf{x}) - e_{lmn}\widehat{I}_{lji}(\mathbf{x}) \tag{A.1a}$$

$$L_{mnji}(\mathbf{x}) = J_{nji,m}(\mathbf{x}) - e_{lmn}\widehat{J}_{lji}(\mathbf{x}) \tag{A.1b}$$

$$\widehat{S}_{mnji}(\mathbf{x}) = \widehat{I}_{nji,m}(\mathbf{x}) \tag{A.1c}$$

$$\widehat{K}_{mnji}(\mathbf{x}) = \widehat{I}_{nji,m}(\mathbf{x}) \tag{A.1d}$$

where

$$I^S_{nji} = \frac{\lambda+\mu}{\lambda+2\mu}\psi_{,ijn}(\mathbf{x}) - \frac{\lambda}{\lambda+2\mu}\delta_{ij}\phi_{,n}(\mathbf{x}) - \delta_{in}\phi_{,j}(\mathbf{x}) - \delta_{jn}\phi_{,i}(\mathbf{x})$$

$$I_{nji} = 2Ph^2\mu\phi_{,ijn}(\mathbf{x}) + \frac{2\kappa}{\mu}[\delta_{jn}\phi_{,i}(\mathbf{x}) - \delta_{in}\phi_{,j}(\mathbf{x})] - Ph^2[\lambda\delta_{ij}M_{,kkn}(\mathbf{x},h) + 2\mu M_{,ijn}(\mathbf{x},h)] + P\lambda\delta_{ij}M_{,n}(\mathbf{x},h) + \left[P(\mu+\kappa)+\frac{\kappa}{\mu}\right]\delta_{in}M_{,j}(\mathbf{x},h) + \left[P(\mu-\kappa)-\frac{\kappa}{\mu}\right]\delta_{jn}M_{,i}(\mathbf{x},h)$$

$$J_{nji}(\mathbf{x}) = -\frac{1}{2\mu}[(\beta+\gamma)e_{nik}\phi_{,jk}(\mathbf{x}) + (\beta-\gamma)e_{njk}\phi_{,ik}(\mathbf{x})] + \frac{1}{2\mu}[(\beta+\gamma)e_{nik}M_{,jk}(\mathbf{x},h) + (\beta-\gamma)e_{njk}M_{,ik}(\mathbf{x},h)]$$

$$\widehat{I}_{nji}(\mathbf{x}) = \frac{1}{2\mu}[\kappa e_{ijk}\phi_{,kn}(\mathbf{x}) - (\mu+\kappa)e_{nik}\phi_{,kj}(\mathbf{x}) - (\mu-\kappa)e_{njk}\phi_{,ki}(\mathbf{x})] - \frac{1}{2\mu}[(\mu+\kappa)e_{ijk}M_{,kn}(\mathbf{x},h) - (\mu+\kappa)e_{nik}M_{,kj}(\mathbf{x},h) - (\mu-\kappa)e_{njk}M_{,ki}(\mathbf{x},h)] + \frac{1}{2}e_{ijk}M_{,kn}(\mathbf{x},g) + \frac{\mu+\kappa}{2\mu h^2}e_{ijn}M(\mathbf{x},h)$$

$$\widehat{J}_{nji}(\mathbf{x}) = -\frac{\beta}{2\mu}\phi_{,ijn}(\mathbf{x}) + \frac{\mu+\kappa}{4\mu\kappa}[\alpha\delta_{ij}M_{,kkn}(\mathbf{x},h) + 2\beta M_{,ijn}(\mathbf{x},h)] - \frac{1}{4\kappa}[\alpha\delta_{ij}M_{,kkn}(\mathbf{x},g) + 2\beta M_{,ijn}(\mathbf{x},g)] - \frac{\mu+\kappa}{4\mu\kappa h^2}[\alpha\delta_{ij}M_{,n}(\mathbf{x},h) + (\beta+\gamma)\delta_{in}M_{,j}(\mathbf{x},h) + (\beta-\gamma)\delta_{jn}M_{,i}(\mathbf{x},h)]$$

The constants introduced in the previous equations are defined by

$$P = \kappa/[\mu(\mu+\kappa)], \quad h^2 = \frac{(\mu+\kappa)(\gamma+\beta)}{4\mu\kappa}, \quad g^2 = \frac{(\alpha+2\beta)}{4\kappa}$$

It is seen that evaluation of the micropolar Eshelby tensors depends on the following three potential functions and their derivatives, which are defined by

$$\psi(\mathbf{x}) = \frac{1}{4\pi}\int_\Omega x\,\mathrm{d}\mathbf{x}', \quad \phi(\mathbf{x}) = \frac{1}{4\pi}\int_\Omega \frac{1}{x}\mathrm{d}\mathbf{x}', \quad M(\mathbf{x},k) = \frac{1}{4\pi}\int_\Omega \frac{\mathrm{e}^{-x/k}}{x}\mathrm{d}\mathbf{x}' \tag{A.2}$$

where $x = |\mathbf{x}|$.

The first and second integrals appeared in Eq. (A.2) are the same as in classical Eshelby tensor (Mura, 1982), and they have been evaluated analytically for a general ellipsoidal inclusion. For a micropolar material, the last integral in Eq. (A.2) cannot be evaluated in a complete analytical form for a general ellipsoidal inclusion, however the potential $M(\mathrm{x}, k)$ can be reduced to the following one-dimensional integral (Ma and Hu, submitted):

$$M(\mathbf{x},k) = \frac{1}{4\pi}\int_\Omega \frac{\mathrm{e}^{-x/k}}{x}\mathrm{d}\mathbf{x}' = k^2 - k^2\frac{a_3}{2}\int_0^\infty (D\times A)\mathrm{d}u \tag{A.3}$$

where the constants in Eq. (A.3) are defined as

$$D = \frac{1}{(u+a_3^2)^{3/2}}\left(1+\frac{a}{k}\sqrt{\frac{u+a_3^2}{u+a^2}}\right)\exp\left(-\frac{a}{k}\sqrt{\frac{u+a_3^2}{u+a^2}}\right)$$

$$A = I_0(B\rho)\cosh(Cz), \quad B = \frac{1}{k}\sqrt{\frac{u}{u+a^2}}, \quad C = \frac{a}{k\sqrt{u+a^2}}$$

$$u = a_3^2\tan^2\theta, \quad \rho = \sqrt{x_1^2 + x_2^2}$$

I_M is the Mth order modified Bessel function of the first kind, a is the radius of the short axis of the ellipsoidal and a_3 is its radius of the major axis. The major axis of the ellipsoidal lines with the axis z. The derivatives of Eq. (A.3) are given by

$$M_{,i}(\mathbf{x},k) = -\frac{a_3}{2}k^2\int_0^\infty (D \times A_{,i})\mathrm{d}u \tag{A.4a}$$

$$M_{,ij}(\mathbf{x},k) = -\frac{a_3}{2}k^2\int_0^\infty (D \times A_{,ij})\mathrm{d}u \tag{A.4b}$$

$$M_{,ijm}(\mathbf{x},k) = -\frac{a_3}{2}k^2\int_0^\infty (D \times A_{,ijm})\mathrm{d}u \tag{A.4c}$$

$$M_{,ijmn}(\mathbf{x},k) = -\frac{a_3}{2}k^2\int_0^\infty (D \times A_{,ijmn})\mathrm{d}u \tag{A.4d}$$

where

$$A_{,\alpha} = B\cosh(Cz)I_1(B\rho)\frac{x_\alpha}{\rho}$$

$$A_{,\alpha\beta} = B\cosh(Cz)\frac{1}{2\rho^3}[B\rho[I_0(B\rho) + I_2(B\rho)]x_\alpha x_\beta + 2I_1(B\rho)(\rho^2\delta_{\alpha\beta} - x_\alpha x_\beta)]$$

$$\begin{aligned}A_{,\alpha\beta\gamma} = B\cosh(Cz)\Big\{&\left[-\frac{3B}{2\rho^4}x_\alpha x_\beta x_\gamma + \frac{B}{2\rho^2}\left(\delta_{\alpha\beta}x_\gamma + \delta_{\alpha\gamma}x_\beta + \delta_{\gamma\beta}x_\alpha\right)\right]I_0(B\rho)\\ &+\left[\frac{3}{\rho^5}x_\alpha x_\beta x_\gamma + \frac{3B^2}{4\rho^3}x_\alpha x_\beta x_\gamma - \frac{1}{\rho^3}\left(\delta_{\alpha\beta}x_\gamma + \delta_{\alpha\gamma}x_\beta + \delta_{\gamma\beta}x_\alpha\right)\right]I_1(B\rho)\\ &+\left[-\frac{3B}{2\rho^4}x_\alpha x_\beta x_\gamma + \frac{B}{2\rho^2}\left(\delta_{\alpha\beta}x_\gamma + \delta_{\alpha\gamma}x_\beta + \delta_{\gamma\beta}x_\alpha\right)\right]I_2(B\rho) + \left[\frac{B^2}{4\rho^3}x_\alpha x_\beta x_\gamma\right]I_3(B\rho)\Big\}\end{aligned}$$

$$\begin{aligned}A_{,\alpha\beta\gamma\lambda} = B\cosh(Cz)\Big\{&\frac{B}{\rho^6}\Big[\frac{15}{2}x_\alpha x_\beta x_\gamma x_\lambda + \frac{3}{8}B^2\rho^2 x_\alpha x_\beta x_\gamma x_\lambda - \frac{3}{2}\rho^2 x_\lambda\left(\delta_{\alpha\beta}x_\gamma + \delta_{\alpha\gamma}x_\beta + \delta_{\gamma\beta}x_\alpha\right)\\ &-\frac{3}{2}\rho^2\left(\delta_{\alpha\lambda}x_\beta x_\gamma + \delta_{\beta\lambda}x_\alpha x_\gamma + \delta_{\gamma\lambda}x_\beta x_\alpha\right) + \frac{\rho^4}{2}\left(\delta_{\alpha\beta}\delta_{\gamma\lambda} + \delta_{\alpha\gamma}\delta_{\beta\lambda} + \delta_{\alpha\lambda}\delta_{\gamma\beta}\right)\Big]I_0(B\rho)\\ &+\frac{1}{\rho^7}\Big[-\frac{9}{2}B^2\rho^2 x_\alpha x_\beta x_\gamma x_\lambda - 15x_\alpha x_\beta x_\gamma x_\lambda + \frac{3}{4}B^2\rho^4 x_\lambda\left(\delta_{\alpha\beta}x_\gamma + \delta_{\alpha\gamma}x_\beta + \delta_{\gamma\beta}x_\alpha\right)\\ &+3\rho^2 x_\lambda\left(\delta_{\alpha\beta}x_\gamma + \delta_{\alpha\gamma}x_\beta + \delta_{\gamma\beta}x_\alpha\right) + 3\rho^2\left(\delta_{\alpha\lambda}x_\beta x_\gamma + \delta_{\beta\lambda}x_\alpha x_\gamma + \delta_{\gamma\lambda}x_\beta x_\alpha\right)\\ &+\frac{3}{4}B^2\rho^4\left(\delta_{\alpha\lambda}x_\beta x_\gamma + \delta_{\beta\lambda}x_\alpha x_\gamma + \delta_{\gamma\lambda}x_\beta x_\alpha\right) - \rho^4\left(\delta_{\alpha\beta}\delta_{\gamma\lambda} + \delta_{\alpha\gamma}\delta_{\beta\lambda} + \delta_{\alpha\lambda}\delta_{\gamma\beta}\right)\Big]I_1(B\rho)\\ &+\frac{B}{\rho^6}\Big[\frac{15}{2}x_\alpha x_\beta x_\gamma x_\lambda + \frac{B^2\rho^2}{2}x_\alpha x_\beta x_\gamma x_\lambda - \frac{3}{2}\rho^2 x_\lambda\left(\delta_{\alpha\beta}x_\gamma + \delta_{\alpha\gamma}x_\beta + \delta_{\gamma\beta}x_\alpha\right)\end{aligned}$$

$$-\frac{3}{2}\rho^2\left(\delta_{\alpha\lambda}x_\beta x_\gamma+\delta_{\beta\lambda}x_\alpha x_\gamma+\delta_{\gamma\lambda}x_\beta x_\alpha\right)+\frac{\rho^4}{2}\left(\delta_{\alpha\beta}\delta_{\gamma\lambda}+\delta_{\alpha\gamma}\delta_{\beta\lambda}+\delta_{\alpha\lambda}\delta_{\gamma\beta}\right)\Big]I_2(B\rho)$$
$$+\frac{B^2}{\rho^5}\left[-\frac{3}{2}x_\alpha x_\beta x_\gamma x_\lambda+\frac{\rho^2}{4}x_\lambda\left(\delta_{\alpha\beta}x_\gamma+\delta_{\alpha\gamma}x_\beta+\delta_{\gamma\beta}x_\alpha\right)+\frac{\rho^2}{4}\left(\delta_{\alpha\lambda}x_\beta x_\gamma+\delta_{\beta\lambda}x_\alpha x_\gamma+\delta_{\gamma\lambda}x_\beta x_\alpha\right)\right]I_3(B\rho)$$
$$+\frac{B^3}{\rho^4}\left[\frac{1}{8}x_\alpha x_\beta x_\gamma x_\lambda\right]I_4(B\rho)\Big\}$$

The symbols α, β, γ, λ range from 1 to 2, and

$$A_{,z}=C\sinh(Cz)I_0(B\rho)$$
$$A_{,zz}=C^2\cosh(Cz)I_0(B\rho),\quad A_{,\alpha z}=(A_{,\alpha})_{,z}$$
$$A_{,zzz}=C^3\sinh(Cz)I_0(B\rho),\quad A_{,\alpha zz}=(A_{,\alpha})_{,zz},\quad A_{,\alpha\beta z}=\left(A_{,\alpha\beta}\right)_{,z}$$
$$A_{,zzzz}=C^4\cosh(Cz)I_0(B\rho),\quad A_{,\alpha zzz}=(A_{,\alpha})_{,zzz},\quad A_{,\alpha\beta zz}=\left(A_{,\alpha\beta}\right)_{,zz},\quad A_{,\alpha\beta\gamma z}=\left(A_{,\alpha\beta\gamma}\right)_{,z}$$

With the previous expression, the micropolar Eshelby tensors and their average over the ellipsoidal domain can be calculated.

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